

Thermodynamics of Multi-Component Fermi Vapors

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Abstract

We study the thermodynamical properties of Fermi vapors confined in a harmonic external potential. In the case of the ideal Fermi gas, we compare exact density profiles with their semiclassical approximation in the conditions of recent experiments. Then, we consider the phase-separation of a multi-component Fermi vapor. In particular, we analyze the phase-separation as a function of temperature, number of particles and scattering length. Finally, we discuss the effect of rotation on the stability and thermodynamics of the trapped vapors.

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I. INTRODUCTION

Recent experiments with dilute vapors of alkali-metal atoms in magnetic or magneto-optical traps at very low temperatures have renewed the theoretical study of Bose and Fermi gases. For atomic gases, the Bose-Einstein condensation has been achieved in 1995 [1-3] and the Fermi quantum degeneracy in 1999 [4,5].

The s-wave scattering between Fermions in the same hyperfine state is inhibited due the Pauli principle. It follows that at low temperature the dilute Fermi gas, in a fixed hyperfine state, is practically ideal. Nevertheless, the effect of interaction could be very effective for a Fermi vapor with two or more hyperfine states (components). In the recent experiment with dilute ^{40}K Fermions [4,5], to favor the evaporative cooling, a ^{40}K Fermi vapor in two hyperfine states ($|9/2, 9/2\rangle$ and $|9/2, 7/2\rangle$) is used. When the system is below the Fermi temperature, one hyperfine component is removed and it remains a trapped quasi-ideal degenerate Fermi gas.

In this paper we analyze the thermodynamical properties of a many-component interacting Fermi vapor by using a harmonic potential that models the trap of recent experiment with ^{40}K [4,5]. These trapped Fermi gases are quite interesting because the quantum degeneracy shows up not only in momentum space, as in uniform systems, but also in coordinate space. First, we investigate the accuracy of the semiclassical approximation for the study of the properties of the Fermi gas in the conditions of ^{40}K experiment. For the ideal Fermi gas we discuss the finite-temperature spatial and momentum distribution and derive, in the large impulse approximation, the dynamic structure factor. Then, we consider Fermi vapors with many components. Due to the nonzero interaction between different hyperfine states, one can obtain the phase-separation of Fermi components by varying temperature, number of particles and scattering length. Finally, we discuss the effect of an imposed rotation around a fixed axis on stability and thermodynamics of the trapped Fermi vapors.

II. IDEAL FERMI GAS

For an ideal Fermi gas at thermal equilibrium, the average number of particles in the single-particle state $|\alpha\rangle$ with energy ϵ_α is given by

$$n_\alpha = \frac{1}{e^{\beta(\epsilon_\alpha - \mu)} + 1} , \quad (1)$$

where μ is the chemical potential and $\beta = 1/(kT)$ with k the Boltzmann constant and T the temperature. The average number $N = \sum_\alpha n_\alpha$ of particles of the system fixes the chemical potential [6]. In the case of a harmonic potential $U(\mathbf{r}) = (m/2)(\omega_1^2 x_1^2 + \omega_2^2 x_2^2 + \omega_3^2 x_3^2)$ one finds the exact density profile by using the Eq. (1) and Hermite polynomials, from which one obtains the eigenfunctions $\phi_{n_1 n_2 n_3}(\mathbf{r})$ of the harmonic oscillator

$$n(\mathbf{r}) = \sum_{n_1 n_2 n_3=0}^{\infty} \frac{|\phi_{n_1 n_2 n_3}(\mathbf{r})|^2}{e^{\beta \hbar (\omega_1(n_1+1/2) + \omega_2(n_2+1/2) + \omega_3(n_3+1/2) - \mu)} + 1} . \quad (2)$$

The exact density profile can be compared with the semiclassical one. The semiclassical approximation means that, instead of ϵ_α , one uses the classical single-particle phase-space energy $\epsilon(\mathbf{r}, \mathbf{p}) = \frac{\mathbf{p}^2}{2m} + U(\mathbf{r})$. In this way one obtains the single-particle phase-space distribution

$$n(\mathbf{r}, \mathbf{p}) = \frac{1}{e^{\beta(\epsilon(\mathbf{r}, \mathbf{p}) - \mu)} + 1} . \quad (3)$$

At zero temperature, i.e., in the limit $\beta \rightarrow \infty$ where $\mu = E_F$ (the Fermi energy), one has $n(\mathbf{r}, \mathbf{p}) = \Theta(E_F - \epsilon(\mathbf{r}, \mathbf{p}))$, where $\Theta(x)$ is the Heaviside step function. At temperature well below the Fermi temperature $T_F = E_F/k$, the Fermions begin to fill the lowest available single-particle states in accordance with the Pauli exclusion Principle (Fermi quantum degeneracy) [6].

Because of the Heisenberg principle, the quantum elementary volume of the single-particle phase-space is given by $(2\pi\hbar)^3$, where \hbar is the Planck constant. It follows that the average number N of particles can be written as

$$N = \int \frac{d^3\mathbf{r} d^3\mathbf{p}}{(2\pi\hbar)^3} n(\mathbf{r}, \mathbf{p}) = \int d^3\mathbf{r} n(\mathbf{r}) = \int d^3\mathbf{p} \tilde{n}(\mathbf{p}) , \quad (4)$$

where $n(\mathbf{r})$ is the spatial distribution and $\tilde{n}(\mathbf{p})$ is the momentum distribution. The finite temperature spatial distribution is given by

$$n(\mathbf{r}) = \frac{1}{\lambda^3} f_{3/2} \left(e^{\beta(\mu - U(\mathbf{r}))} \right), \quad (5)$$

where $\lambda = (2\pi\hbar^2\beta/m)^{1/2}$ is the thermal length and

$$f_n(z) = \frac{1}{\Gamma(n)} \int_0^\infty dx \frac{x^{n-1}}{z^{-1}e^x + 1}, \quad (6)$$

with Γ the factorial function. In the limit of zero temperature, with $\mu = E_F$ the Fermi energy, the spatial distribution gives the Thomas-Fermi approximation $n(\mathbf{r}) = (2m)^{3/2}/(6\pi^2\hbar^3) (E_F - U(\mathbf{r}))^{3/2} \Theta(E_F - U(\mathbf{r}))$, where Θ is the Heaviside step function [7-8]. The Eq. (5) is the generalization of well-known formula for an ideal homogenous Fermi gas in a box that is exact in the thermodynamical limit [6]. Note that for $|z| < 1$ one has $f_n(z) = \sum_{i=1}^\infty (-1)^{i+1} z^i / i^n$. Moreover, by using $g_n(z) = -f_n(-z)$ instead of $f_n(z)$, one finds the spatial distribution of the ideal Bose gas in external potential.

A comparison between exact and semiclassical results has been recently performed at zero [9] and finite temperature [10]. It has been shown that the semiclassical approximation is good for $(kT/\hbar\omega_H) \gg 1$, where $\omega_H = (\omega_1\omega_2\omega_3)^{1/3}$, or, at a fixed temperature, for a large number N of trapped particles. Our calculations confirm this prediction. In fact, we find that when $(kT/\hbar\omega_H) > 1$ there are no appreciable deviations between exact and semiclassical results. Instead, when $(kT/\hbar\omega_H) < 1$ some differences are observable, in particular for "magic" numbers of particles ($N = 1, 4, 10, 20, 35, 56, 84, \dots$) that correspond to a complete shell occupation of single-particle energy levels (see also [10]). The differences are reduced by increasing N showing that semiclassical approximation provides an excellent representation of Fermi distribution for a wide range of parameters. In Figure 1 we plot the exact density profile, obtained by a direct calculation of Eq. (2), and that obtained with the semiclassical function (5) for $(kT/\hbar\omega_H) = 10^{-3}$ and three "magic" numbers. In correspondence of the "magic numbers", the spatial density profile shows local maxima, which suggest a spatial shell structure. The magic numbers are particularly stable; in fact, for small variations of

the chemical potential μ the magic number N remains unchanged. The shell structure in the density profile is washed out by increasing the number of particles and is completely absent in the semiclassical approximation.

As shown in the experiment [4], for the quasi-ideal one-component Fermi gas of ^{40}K atoms confined in a cigar-shaped trap ($\omega_1 = \omega_2 = 860.80$ Hz, $\omega_3 = 122.52$ Hz), the semiclassical formulas of an ideal Fermi gas in a harmonic trap agree with the available experimental data within the statistical uncertainty. The accuracy of the semiclassical predictions are not surprising because the experimental conditions are $kT/(\hbar\omega_H) \simeq 10^2\text{-}10^3$ and $N \simeq 10^6$. For the sake of completeness, in Table 1 we show some relevant quantities of the Fermi gas at different temperatures: the energy $E = 3/(2\beta\lambda^3) \int d^3\mathbf{r} f_{5/2} \left(e^{\beta(\mu - U(\mathbf{r}))} \right) + \int d^3\mathbf{r} U(\mathbf{r}) n(\mathbf{r})$, the density at the origin $n(\mathbf{0})$ and the widths $\langle \rho^2 \rangle^{1/2}$ and $\langle z^2 \rangle^{1/2}$ of the Fermi cloud in the radial and axial directions, respectively.

In addition to the spatial distribution $n(\mathbf{r})$, also the momentum distribution $\tilde{n}(\mathbf{p})$ gives an useful signature of the quantum degeneracy of the Fermi gas. In the semiclassical approximation, the finite temperature momentum distribution of an ideal Fermi gas in a harmonic trap is calculated from Eq. (4) and reads

$$\tilde{n}(\mathbf{p}) = \frac{1}{(m\omega_H\lambda)^3} f_{3/2} \left(e^{\beta(\mu - \frac{\mathbf{p}^2}{2m})} \right). \quad (7)$$

At zero temperature, one recovers the formula found by Butts and Rokhsar [7], namely $\tilde{n}(\mathbf{p}) = 1/(6\pi^2\hbar^3) 2^{3/2}/(m\omega_H^2)^{3/2} (E_F - \mathbf{p}^2/2m)^{3/2} \Theta(E_F - \mathbf{p}^2/2m)$. Despite the spatial anisotropy of the trap, the momentum distribution of the Fermi gas is isotropic when the semiclassical approximation is used. Moreover, it is not difficult to show that, both in the exact and semiclassical cases, the momentum distribution \tilde{n} can be found from the spatial one n by the simple relation $\tilde{n}(p_1, p_2, p_3) = n(p_1/(m\omega_1), p_2/(m\omega_2), p_3/(m\omega_3)) / (m\omega_H)^3$, where $\mathbf{p} = (p_1, p_2, p_3)$. In general, the exact momentum distribution is not isotropic and it presents the same shell structures of the spatial one when N is close to a magic number. Finally, the Eq. (7) can be used to calculate the dynamic structure factor $S(\mathbf{q}, E)$ of the Fermi gas. In the large impulse approximation [11] one finds the remarkable result

$$S(\mathbf{q}, E) = \frac{2\pi m^2}{q\beta} \frac{1}{(m\omega_H\lambda)^3} f_{5/2} \left(e^{\beta \left(\mu - \frac{(E - \mathbf{q}^2/2m)^2 m}{2\mathbf{q}^2} \right)} \right), \quad (8)$$

where \mathbf{q} and E are the momentum and energy transferred by the probe to the sample. Recently, the dynamic structure factor of a trapped Bose-Einstein condensate has been measured using two-photon optical Bragg spectroscopy with a time of flight technique by which the number of optically excited atoms can be counted [12].

III. MULTI-COMPONENT FERMİ VAPOR

The problem of a dilute Fermi vapor with M hyperfine states (components) can be studied by using the s-wave scattering approximation, the mean-field approach and semiclassical formulas. The spatial density profile $n_i(\mathbf{r})$ of the i -th component of a Fermi vapor can be written as

$$n_i(\mathbf{r}) = \frac{1}{\lambda^3} f_{3/2} \left(e^{\beta \left(\mu_i - U(\mathbf{r}) - \sum_{j=1}^M g_{ij} n_j(\mathbf{r}) \right)} \right), \quad (9)$$

where $i = 1, 2, \dots, M$, μ_i is the chemical potential of the i -th component, and $g_{ij} = 4\pi\hbar^2 a_{ij}/m$, with a_{ij} the s-wave scattering length between i -th and j -th component ($a_{ii} = 0$). Thus, the effect of the other $M - 1$ Fermi components on the i -th component is the appearance of a mean-field effective potential. Note that in the limit of zero temperature, one finds the equations used by Amoruso *et al* [13]. The total energy of a Fermi vapor with M components is given by

$$E = \sum_{i=1}^M \frac{3}{2} \frac{1}{\beta\lambda^3} \int d^3\mathbf{r} f_{5/2} \left(e^{\beta \left(\mu_i - U(\mathbf{r}) - \sum_{j=1}^M g_{ij} n_j(\mathbf{r}) \right)} \right) + \sum_{i=1}^M \int d^3\mathbf{r} U(\mathbf{r}) n_i(\mathbf{r}) + \sum_{i < j=1}^M \int d^3\mathbf{r} g_{ij} n_i(\mathbf{r}) n_j(\mathbf{r}), \quad (10)$$

where the first term is the kinetic energy E_{kin} , the second term is the external potential energy E_{ext} and the third term is the interaction energy E_{int} . At zero temperature the kinetic energy assumes the familiar Thomas-Fermi form $E_{kin} = \sum_{i=1}^M (6\pi^2)^{2/3} (3\hbar^2)/(5m) \int d^3\mathbf{r} [n_i(\mathbf{r})]^{5/3}$.

We numerically solve the set of equations (9) with a self-consistent iterative procedure. Recently, we have used a similar iterative procedure to study ^7Li Bose gas [14]. If the components of the Fermi vapor are non-interacting then they can occupy the same spatial region. Instead, if the interaction is strong enough (repulsive interaction) or for N very large one finds a phase-separation, i.e., the Fermi components stay in different spatial regions. This effect, that has been first discussed at $T = 0$ by Amoruso *et al* [13], is shown in Figure 2, where we plot the density profiles of two components of the ^{40}K Fermi vapor for different values of the scattering length a ($a=a_{12}$) and $N = 0.5 \cdot 10^7$ atoms in each component.

When two components have the same number of particles, the onset of phase-separation is also an example of spontaneous symmetry breaking. In fact, if the chemical potentials μ_i of the two components are equal, Eq. (9) always admits a symmetric solution $n_1(\mathbf{r}) = n_2(\mathbf{r})$. However, for particle number N larger than a threshold N_c the solution bifurcates and a pair of symmetry breaking solutions appears. Just beyond threshold the asymptotic solutions begin to differ from the symmetric one in a neighborhood of the origin $\mathbf{r} = \mathbf{0}$, i.e. at the point of higher density. Therefore, an analytic formula for the critical chemical potential μ_c can be obtained by standard bifurcation analysis of Eq. (9) at the origin, which gives

$$\frac{g\beta}{\lambda^3} e^{\beta(\mu - gn(\mathbf{0}))} f'_{3/2}(e^{\beta(\mu - gn(\mathbf{0}))}) = 1 . \quad (11)$$

At $T = 0$, by the use of the first term of the large z expansion of $f_{3/2}(z)$ [6], one finds analytical expressions for the critical density $n_c(\mathbf{0})$ and the critical chemical potential μ_c :

$$n_c(\mathbf{0}) = \frac{\pi}{48a^3} , \quad \mu_c = \frac{5\pi^2}{24} \hbar\omega_H \left(\frac{a_H}{a} \right)^2 . \quad (12)$$

These remarkably simple formulas can be very useful to determine the onset of phase-separation in future experiments. Moreover, by knowing the critical chemical potential one numerically finds the number of particles via Eq. (9).

At finite temperature, we numerically solve the Eq. (11). In Figure 3 we plot the critical number of Fermions as a function of the temperature for different values of the scattering length. Figure 3 shows that, by using a realistic scattering length [4,5], one needs

about $N = 10^{12}$ atoms to get the spontaneous symmetry breaking. One sees that at higher temperatures the phase-separation appears with a larger number of particles or a larger scattering length. In the lower part of Figure 3 we plot the same critical line as a function of temperature for different values of the scattering length. The conclusion is that by increasing the interaction between the two components one can use lower number of particles to obtain the phase-separation. Such a behavior can be extracted from Eq. (12) in the case of zero temperature. Note that at finite temperature one can use the first two terms of the large z expansion of $f_{3/2}(z)$ [6]. Under the conditions $kT \ll \hbar\omega_H$ and $\mu \gg gn(\mathbf{0})$ one finds the following equation for the onset of phase-separation

$$\left(\frac{a}{a_H}N^{1/6}\right)\left[1 - \frac{\pi^2}{2^{11/3}3^{5/3}}\frac{1}{N^{2/3}}\left(\frac{kT}{\hbar\omega_H}\right)^2\right] = \frac{\pi}{2^{5/3}3^{1/6}}. \quad (13)$$

One important point is to understand what happens when the two components of the Fermi vapor have a different number of atoms. In such a case, it is still possible to observe a phase-separation of the two components and their position is related to the ratio N_1/N_2 , where N_1 and N_2 are the number of Fermions in the two hyperfine states. In Figure 4 we plot the density profiles of the two components with different scattering length and ratios N_1/N_2 . The component with less atoms is pushed outward but this effect appears only when the scattering length exceeds the value that produces the onset of phase separation with $N_1 = N_2$.

Phase-separation also appears in a Fermi vapor with three or more components. In Figure 5 we plot the density profiles of the ^{40}K Fermi vapor with three components. In this case we numerically solve Eq. (9) with $a_{12} = a_{13} = a_{23} = a$. The Figure shows that also for three components the spontaneous symmetry breaking and the phase-separation are controlled by scattering length, temperature and total number of particles. In particular, one finds that by increasing the scattering length at first one of the components separates from the others, which remain still mixed. Note that the separation begins at the center of the trap. As the scattering length is further increased, also these two components separate (this second phase-separation begins at the interface with the previously separated component) and one

eventually sees complete phase-separation and the formation of 4 or 5 shells.

The Eq. (11) can be extended to a M -component Fermi vapor with the same number of particles in each component. The critical density $n_c(\mathbf{0})$ does not depend on the number M of Fermi components and one gets the same result of Eq. (12). Instead, the critical chemical potential reads $\mu_c = (2M + 1)\pi^2\hbar\omega_H(a_H/a)^2/24$.

Nowadays it is possible to confine mixtures of Fermions and Bosons. If Bosons occupy the external region of the trap then the internal Fermi gas can increase its density at the origin and so favor the onset of phase-separation. We have verified that such effect is possible when two conditions are satisfied: 1) the number of Bosons is close to that of Fermions; 2) the Boson-Boson interaction is close to the Boson-Fermion one with scattering length of at least 10^3 Bohr radii. Nevertheless the gain is limited: the density at the origin grows by 20% for $N = 10^7$ particles. In conclusion, to obtain the phase-separation with several millions of Fermions, the only chance is to strongly enhance the scattering length, for example by using the Feshbach resonances. The Feshbach resonances, already seen in Bose condensates [15], should allow to vary the interaction strength between atoms. One could use them to go smoothly from a quasi-ideal to a strongly interacting Fermi system.

IV. ROTATING FERMI VAPOR

The semiclassical approximation can be easily applied to a rotating gas. For an ideal gas, that is rotating with angular velocity Ω around the x_3 -axis, the classical single-particle energy is $\epsilon(\mathbf{r}, \mathbf{p}) = \frac{\mathbf{p}^2}{2m} + U(\mathbf{r}) - \Omega(x_1p_2 - x_2p_1)$. One immediately finds that the spatial distribution of an ideal Fermi gas is given by

$$n(\mathbf{r}) = \frac{1}{\lambda^3} f_{3/2} \left(e^{\beta(\mu - U(\mathbf{r}) + \frac{1}{2}m\Omega^2\rho^2)} \right), \quad (14)$$

where $\rho = (x_1^2 + x_2^2)^{1/2}$. Note that the effective potential of the rotating ($\Omega > 0$) Fermi gas is $U(\mathbf{r}) - (m/2)\Omega\rho^2$. If $U(\mathbf{r}) \sim \rho^s$ with $s < 2$ for $\rho \rightarrow \infty$ then the effective potential goes to $-\infty$ for $\rho \rightarrow \infty$ and the spatial integral of the density profile is divergent. It follows that

for $s < 2$ the system is unstable against rotation. The same result applies for ideal Bose and Boltzmann gases. Obviously, the Fermi gas can remain confined near the center of the trap for a very long time depending on the tunneling probability.

In the case of the harmonic potential, $s = 2$ and the gas is stable against rotation until Ω is smaller than the smaller of the harmonic frequencies ω_1 and ω_2 . The effect of the rotation is simply a shift in two trap frequencies: $\omega_1 \rightarrow (\omega_1^2 - \Omega^2)^{1/2}$ and $\omega_2 \rightarrow (\omega_2^2 - \Omega^2)^{1/2}$. In this way one gets the Fermi energy of the rotating ideal gas

$$E_F(\Omega) = 6^{1/3} \hbar \omega_H N^{1/3} \left(1 - \frac{\Omega^2}{\omega_1^2}\right)^{1/6} \left(1 - \frac{\Omega^2}{\omega_2^2}\right)^{1/6}. \quad (15)$$

A similar formula has been found in [16] for the Bose-Einstein transition temperature of a Bose gas in axial-symmetric harmonic potential. Also the density profile of the rotating quantum gas is modified according to the frequency shift discussed above.

The discussion can be extended to a Fermi vapor with two or more components. In particular, one can use Eq. (9) and all the other equations by simply shifting the trap frequencies. Such a shift determines an enhancement of the characteristic length of the harmonic trap: $a_H \rightarrow a_H / \left[(1 - \Omega^2/\omega_1^2)^{1/12} (1 - \Omega^2/\omega_2^2)^{1/12}\right]$. It means that the rotating gas is more dilute and consequently the interatomic interaction is less effective. As shown in Eq. (12) and Eq. (13), the rotation increases the critical number of Fermions that are necessary to get phase-separation.

V. CONCLUSIONS

First of all we have analyzed the accuracy of the semiclassical approximation. We have found that the semiclassical approach is very accurate for $kT > \hbar\omega_H$ or, at a fixed temperature, for a large number of particles. In particular, the semiclassical formulas are extremely reliable to study recent experiments with a trapped quasi-ideal ^{40}K Fermi gas where $kT/(\hbar\omega_H) \simeq 10^2$ - 10^3 and $N \simeq 10^6$. By using the semiclassical approach, we have analyzed the spatial and momentum distributions of an ideal Fermi gas in harmonic potential. In par-

ticular, we have derived, in the large impulse approximation, the dynamic structure factor of the system.

Then, we have studied the finite-temperature density profiles of a Fermi vapor with many hyperfine states. In the two-component case, by using the actual scattering length, we have found that the two components occupy the same spatial region. We have shown that the onset of phase-separation appears by increasing the scattering length or, for a fixed scattering length, by increasing the number of particles. By raising the temperature, a larger scattering length or a larger number of particles is needed to obtain the phase-separation. A Fermi vapor with three or more components has the same behavior but at first only one of the components separates from the others, which remain still mixed. The critical density of Fermions at the origin, which gives rise to the phase-separation, does not depend on the number of Fermi component and on the properties of the trap; moreover it satisfies the equation $n_c(\mathbf{0}) = \pi/(48a^3)$, where a is the s-wave scattering length.

Finally, we have considered the Fermi vapor is under rotation around a fixed axis with frequency Ω . We have shown that, in the case of harmonic external potential, the rotation decreases the effective trap frequencies. As a consequence, the threshold for the onset of phase-separation occurs at larger numbers of Fermions.

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T	$E/(kN)$	$\langle \rho^2 \rangle^{1/2}$	$\langle \rho^2 \rangle^{1/2}$	$n(\mathbf{0})$
0.05	0.44	9.10	44.60	14.12
0.27	0.88	12.89	63.43	7.37
0.55	2.66	17.67	87.26	3.15
0.82	2.48	21.56	106.55	1.76
1.10	3.30	24.84	122.90	1.15

TABLE 1. Some properties of the ideal Fermi gas with $N = 7 \cdot 10^5$ atoms in the harmonic trap ($\omega_1 = \omega_2 = 860.80$ Hz, $\omega_3 = 122.52$ Hz). Temperature T and energy per particle $E/(kN)$ in units μK , lengths in units μm and density $n(\mathbf{0})$ in units 10^{12} cm^{-3} .

FIGURE CAPTIONS

Figure 1: Comparison between exact (full) and semiclassical (dashed) density profiles for an ideal Fermi gas in an isotropic harmonic trap. $kT/(\hbar\omega_H) = 10^{-3}$. N is the number of Fermions. Lengths in units $a_H = (\hbar/m\omega_H)^{1/2}$ and densities in units a_H^{-3} .

Figure 2: Density profiles of the ^{40}K vapor with two components in the anisotropic harmonic trap. $N = 0.5 \cdot 10^7$ atoms for each component. a) $T = 0.1T_F$; b) $T = T_F$; c) $T = 2T_F$ ($T_F = 1.07 \mu\text{K}$). Units as in Figure 1.

Figure 3: Critical number N of particles for the phase-separation of the 2-component ^{40}K vapor as a function of the temperature T . On the top: $a = 157a_0$. On the bottom: $a = 10 \cdot 157a_0$ (full diamond), $a = 50 \cdot 157a_0$ (open triangle), $a = 100 \cdot 157a_0$ (full circle). a_0 is the Bohr radius.

Figure 4: Density profiles of the ^{40}K vapor with two components in the anisotropic harmonic trap at zero temperature. $N_1 = 10^7$ atoms. a) $a = 1000a_0$; b) $a = 2700a_0$ (a_0 is Bohr radius). Units as in Figure 1.

Figure 5: Density profiles of the ^{40}K vapor with three components (solid, dotted and dashed lines) in the isotropic harmonic trap ($\omega_H = 450 \text{ Hz}$). $N = 0.5 \cdot 10^7$ atoms for each component. (a) $T = 0.5T_F$; (b) $T = 2T_F$ ($T_F = 1.07 \mu\text{K}$). Units as in Figure 1.











